

# Simulation study of time evolution Stochastic Bass model

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## ABSTRACT

**In this paper the time evolution of stochastic Bass model have been proposed. The study of innovation diffusion (ID) in a social group is a phenomenon of extensive interest and has been studied across wide ranging disciplines. It attempts to explain the dissemination of new ideas, rumors, news, practices and new products throughout a social system. The paper gives an over view of time dependent evolution of the stochastic Bass model.**

**Keywords: Bass model, Innovation diffusion, Time dependent behavior.**

## 1. INTRODUCTION

Most of the work pertaining to ID models traces back to the framework proposed by Bass (1969) based mathematically on the contagion models. Bass model considers the aggregate first purchase growth of consumer durables introduced with a market potential or „ceiling“. Broadly two channels- Mass media and inter-personal communication play vital role in determining the speed and shape of innovation diffusion patterns in social system.

There have been numerous studies based on a range of assumptions regarding social structure, population characteristics and influence coefficients leading to a variety of ID models (Bernhardt and Mackenzie 1972, Bulte 2000). In case of new product diffusion, future sales are stochastic because of unexpected changes in several factors, such as economic and financial conditions, technological improvements, and competition over market share (Kanniainen et al 2011).

The time dependent behavior of a stochastic system is an area of interest for scholars. The methods available for obtaining probability distribution of a non-deterministic system discussed in the literature deals mostly with the equilibrium distribution. It is worth examining the transient behavior of a stochastic system to get more insight about the system.

The model which we considered is stochastic innovation diffusion model. There are two parameters of interest in this model,  $\alpha$  - a measure of the transmission intensity of the source (coefficient of mass media or innovation) and  $\beta$ , a measure of the propagation intensity through interpersonal contacts (coefficient of imitation or word of mouth). We, then propose to investigate the time dependent behavior of adoption level in a population of adopters. The sample paths have been obtained using Gillespie algorithm.

## 2. INNOVATION DIFFUSION AS A PURE BIRTH PROCESS

ID model as a pure birth process characterized in terms of state-dependent birth rate with explicit time dependence taking place through parameters has been considered.

Defining  $n(t)$  as a Markov process, as the number of adopters of innovation at time  $t$ , the transition probability is

$$P\{n \rightarrow n+1 | n(t), t+\Delta t\} = \lambda_n \Delta t + o(\Delta t)$$

Where

$$\lambda_n = \left[ \alpha + \frac{\beta n(t)}{N} \right] (N - n(t)), n(t) = 0, 1, 2, \dots, N$$

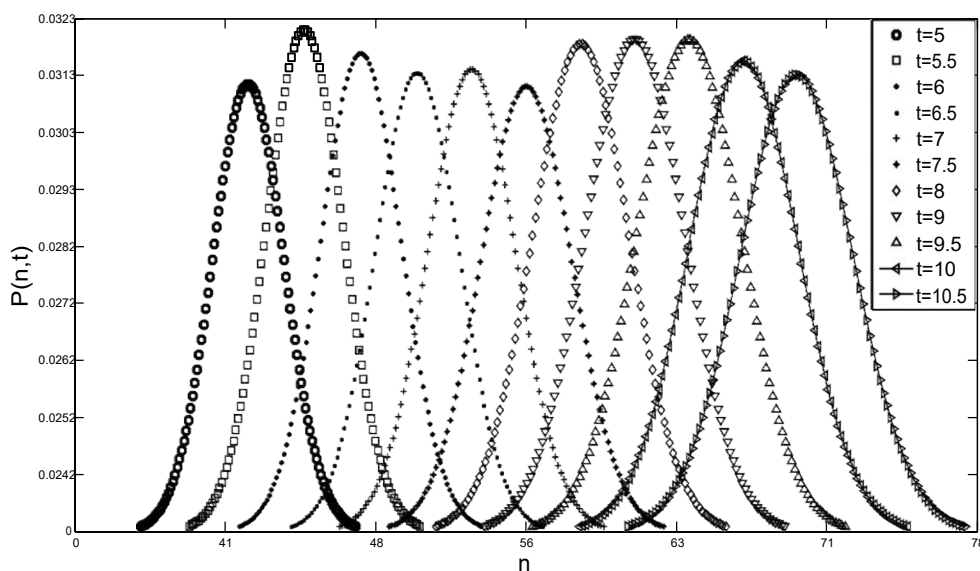
### 3. SIMULATIONS RESULTS AND ANALYSIS

Gillespie proposed an algorithm to simulate Master equation of a system of chemical reactions in a well stirred container.

The algorithm is based on generating two random numbers at each step:

- (i) First random variable is to determine after how much time the next reaction will take place and
- (ii) The second random variable is to choose which one of the reactions will occur.

Mathematically, it is not possible to obtain the time dependent probability distribution of the stochastic ID model. Hence, the simulation has been carried out using Gillespie's stochastic simulation algorithm. The probability distribution is obtained with these sample paths at different time points and is shown in the figure 1.



**Figure1: Probability Distribution at different time points**

### CONCLUSION

Time dependent analysis of the stochastic Bass model has been carried out. The dynamical behavior shows a fluctuating pattern at different intervals of time. The probability distribution of the adopters has been obtained at different time points which give a brief insight about the stochastic time dependent behavior of the ID process.

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